Science and Technology Indonesia
e-ISSN:2580-4391 *p*-ISSN:2580-4405
Vol. 5, No. 1, January 2020

Research Paper

3-Phase Matheuristic Model in Two-Dimensional Cutting Stock Problem of Triangular Shape Items

Putra Bahtera Jaya Bangun [1], Sisca Octarina[1]*, Sisca Puspita Sepriliani [1], Laila Hanum [2], Endro Setyo Cahyono[1]

[1]*Mathematics Department, Faculty of Mathematics and Natural Sciences, Universitas Sriwijaya, Indralaya 30662, South Sumatera, Indonesia*
[2]*Biology Department, Faculty of Mathematics and Natural Sciences, Universitas Sriwijaya, Indralaya 30662, South Sumatera, Indonesia*
***Corresponding author**: sisca_octarina@unsri.ac.id

Abstract
Cutting Stock Problem (CSP) is a problem of cutting stocks with certain cutting rules. This study used the data of rectangular stocks, which cut into triangular shape items with various order sizes. The Modified Branch and Bound Algorithm (MBBA) was used to determine the optimum cutting pattern then formulated it into the 3-Phase Matheuristic model which consisted of constructive phase, improvement phase, and compaction phase. Based on the results, it showed that the MBBA produces three optimum cutting patterns, which was used six times, eight times, and four times respectively to fulfill the consumer demand. Then the cutting patterns were formulated into the 3-Phase Matheuristic model whereas the optimum solution was the minimum trim loss for the first, second and third patterns.

Keywords
Triangular, Modified Branch and Bound Algorithm, Matheuristic

Received: 22 December 2019, Accepted: 18 January 2020
https://doi.org/10.26554/sti.2020.5.1.23-27

1. INTRODUCTION

Raw materials are important in the production process where the material will be converted into desired goods and then sold. Production activities require a variety of raw materials, including paper, wood, yarn, marble and so on. The cutting problem in optimization is known as the Cutting Stock Problem (CSP). CSP is divided into three types namely one-dimensional CSP, two-dimensional CSP, and three-dimensional CSP. These three types of CSP are not only seen from the cutting results but also the residue, which is called trim loss. The smaller of the trim loss obtained, the objective function will be more optimum. Cutting patterns with the smallest trim loss will be used as the optimum cutting pattern.

This research discusses two-dimensional CSP. Rodrigo et al. (2012) created the Pattern Generation algorithm to find cutting patterns. Then, they improved the algorithm to become Modified Branch and Bound Algorithm (Rodrigo et al., 2013). Octarina et al. (2017) explained that in a two-dimensional CSP, the cutting pattern was seen in terms of the length and width of the raw material. CSP is known as cutting raw materials into smaller forms or it also can be interpreted as one of the optimization methods by minimizing the remaining raw materials and maximizing the profits (Rodrigo and Shashikala, 2017). Previous research about two-dimensional CSP has been done, but most of the item was in square or rectangle. Bangun et al. (2019) implemented a branch and cut method on the n-sheet model in solving two-dimensional CSP. Octarina et al. (2018) implemented the Pattern Generation algorithm in forming Gilmore and Gomory model for two-dimensional CSP. Then the research was developed to multiple stock sizes (Octarina et al., 2019).

In this research, we cut the stock into a triangular shape. Cherri et al. (2016) explained that in the 3-Phase Matheuristic model, there were 3 phases including a constructive phase which is useful to get an initial solution, an improvement phase to improve the initial solution and a compaction phase to increase the initial solution to best solution. The 3-Phase Matheuristic model has 2 models namely the Dotted Board model that has been described by Gomes and Oliveira (2006) and the Mixed Integer Linear model that has been described by Toledo et al. (2013). The Dotted Board model is in the constructive and improvement phases. Whereas the Mixed Integer Linear model is in the compaction phase.

This study used data from Rodrigo et al. (2013) that cut raw materials into triangular items of various sizes but they used the Gilmore and Gomory model. Based on this background, this study used the Modified Branch and Bound Algorithm to find cutting patterns then modeled them to a 3-Phase Matheuristic model.

2. EXPERIMENTAL SECTION

2.1 Method

Steps in this research are as follows:

1. Describe the length and the width of the stock includes the side length of triangular items.
2. Define the variables and parameters as follows:
   L is the length of stock, L= 50 cm
   W is the width of stock, W=15 cm
   l_i is the length of item i, where i=1,2,3,4 so l_1=40,25,8,4 cm
   w_i is the width of item i, where i=1,2,3,4 so w_i=13,12,5,2 cm
   e_i is the width of item i, where i=1,2,3,4 so e_i=30,24,2,2 cm
   δ_t^d=0 or 1 whereas 1 if the reference point of item t is positioned in d and 0 if otherwise
   t is the number of item
   d is the positioned of item
3. Find cutting patterns using the Modified Branch and Bound Algorithm
4. Formulate the 3-Phase Matheuristic Model by:
   - Define the objective function to find the minimum initial solution using the Dotted Board Model.
   - Improvise the initial solution using the Dotted Board Model.
   - Get the best solution using the Mixed Integer Linear Model.
5. Solve the 3-Phase Matheuristic Model.

3. RESULTS AND DISCUSSION

3.1 Modified Branch and Bound Algorithm

The data of item size and the number of demand for each item can be seen in Table 1.

Table 1. Item size dan number of demand

Type of Item	1	2	3	4
BC (cm)	40	25	8	4
AD (cm)	13	12	5	2
BD (cm)	30	24	2	2
Demand ((d_i)(*pieces*))	6	30	125	500

These cuts can be categorized as non-oriented cuts, where cuts between the length and width can be reversed. All cutting patterns that were generated from the Modified Branch and Bound Algorithm can be seen in Table 2.

Based on Table 2, there are 28 cutting patterns in the form of triangular items. Next, the optimal pattern will be chosen by looking at a minimal trim loss. The 20th pattern only fulfills the 3rd item and 4th item. So to get the 1st item and 2nd item, the pattern which has a minimum trim loss is taken to produce the item. The optimal pattern can be seen in Table 3.

Based on Table 3. three optimal patterns have a minimal trim loss which can then be used on the model. Furthermore, the 14th cut is called the 1st pattern, the 17th cut is called the 2nd pattern and the 20th cut is called the 3rd pattern. After obtaining the optimal cutting pattern, then the pattern can be made according to the existing cutting pattern. Furthermore, to meet the demand for item 1, the 1st cutting pattern is used. Items 2 are fulfilled by using 6 times of the first pattern and 8 times of the second pattern. Items 3 are fulfilled by using 4 times of the third pattern. Items 4 are fulfilled by using 6 times of the first pattern, 8 times of the second pattern, and 4 times of the third pattern.

Table 2. Cutting Patterns

i^{th}Item	Cutting Pattern						
	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	0	0	0	1	0	1
3	4	8	4	7	3	3	3
4	27	27	63	32	32	71	34
Cut loss (cm^2)	152	168	158	222	152	146	144
i^{th}Item	8	9	10	11	12	13	14
1	1	1	1	1	1	1	1
2	1	0	0	1	0	0	1
3	2	7	6	1	5	1	0
4	44	34	42	51	52	88	59
Cut loss (cm^2)	124	214	202	116	182	118	104
i^{th}Item	15	16	17	18	19	20	21
1	1	0	0	0	0	0	0
2	0	3	3	1	1	0	0
3	0	4	0	19	15	33	27
4	95	25	48	14	39	8	35
Cut loss (cm^2)	110	120	108	164	144	58	70
i^{th}Item	22	23	24	25	26	27	28
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	21	15	10	6	3	1	0
4	63	91	112	125	152	154	172
Cut loss (cm^2)	78	86	102	98	82	82	91

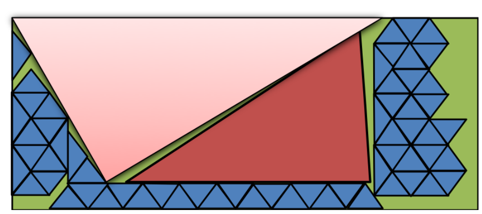

Figure 1. The First Pattern

Figure 1 shows there are 1 piece each of item 1 and item 2 and 59 pieces of item 4. Then, the second cutting pattern on the dotted board can be seen in Figure 2.

Figure 2 shows there are 3 pieces of item 2 and 48 pieces of item 4. The last, the third cutting pattern on the dotted board

Table 3. Optimal Cutting Patterns

i^{th}Item	Optimal Pattern 14	17	20	Demand	Surplus
1	1	0	0	6	0
2	1	3	0	30	0
3	0	0	33	125	7
4	59	48	8	500	270
Cut loss (cm^2)	104	108	58	-	-
Usage	6	8	4	-	-

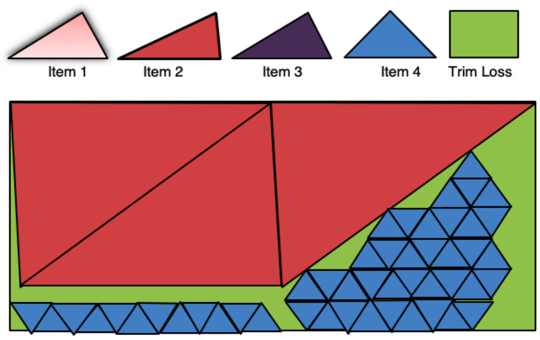

Figure 2. The Second Pattern

can be seen in Figure 3. Figure 3 shows there are 33 pieces of item 3 and 8 pieces of item 4.

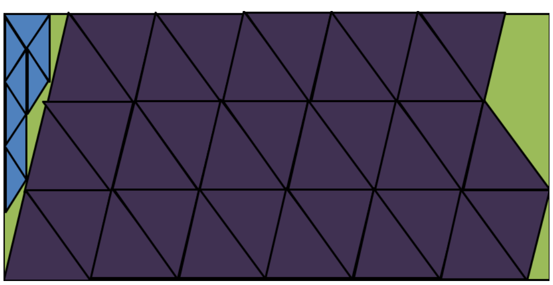

Figure 3. The Third Pattern

3.2 3-phase Matheuristic Model

This formulation has 3 phases including the constructive phase, improvement phase, and compaction phase. This research assumes that item rotation is allowed but the values of l_i, w_i and e_i are assumed not to change even though the item has a rotation. The board used is rectangular with a length of L= 50 cm and a width of W=15 cm (50.15) where there are 4 types of items placed on the board.

3.2.1 3-Phase Matheuristic Model For The First Pattern

The constructive phase for the first pattern can be seen in Model (1).

Minimize

$$\begin{aligned} z = {} & 41.\ \delta_1^{656} + 39.\ \delta_2^{639} + 2.\delta_4^{45} + 8.\delta_4^{74} + 2.\delta_4^{40} + 8.\delta_4^{70} + 12.\delta_4^{96} + 18.\delta_4^{100} + 2.\delta_4^{36} + \\ & 4.\delta_4^{68} + 8.\delta_4^{132} + 8.\delta_4^{128} + 20.\delta_4^{163} + 12.\delta_4^{193} + 14.\delta_4^{227} + 16.\delta_4^{257} + 18.\delta_4^{291} + 20.\delta_4^{321} + \\ & 22.\delta_4^{365} + 24.\delta_4^{395} + 26.\delta_4^{429} + 28.\delta_4^{459} + 30.\delta_4^{493} + 32.\delta_4^{523} + 34.\delta_4^{557} + 36.\delta_4^{587} + \\ & 38.\delta_4^{611} + 40.\delta_4^{641} + 41.\delta_4^{668} + 41.\delta_4^{664} + 41.\delta_4^{660} + 86.\delta_4^{702} + 86.\delta_4^{698} + 86.\delta_4^{694} + \\ & 43.\delta_4^{690} + 45.\delta_4^{736} + 90.\delta_4^{732} + 90.\delta_4^{728} + 90.\delta_4^{724} + 94.\delta_4^{766} + 94.\delta_4^{762} + 94.\delta_4^{758} + \\ & 47.\delta_4^{754} + 98.\delta_4^{788} + 49.\delta_4^{792} \end{aligned} \tag{1}$$

Subject to

$$\delta_1^{656} = 1 \tag{1.a}$$

$$\delta_2^{639} = 1 \tag{1.b}$$

$$\begin{aligned} & \delta_4^{45} + \delta_4^{74} + \delta_4^{40} + \delta_4^{70} + \delta_4^{96} + \delta_4^{100} + \delta_4^{36} + \delta_4^{68} + \delta_4^{132} + \delta_4^{128} + \delta_4^{163} + \delta_4^{193} + \delta_4^{227} + \\ & \delta_4^{257} + \delta_4^{291} + \delta_4^{321} + \delta_4^{365} + \delta_4^{395} + \delta_4^{429} + \delta_4^{459} + \delta_4^{493} + \delta_4^{523} + \delta_4^{557} + \delta_4^{587} + \delta_4^{611} + \\ & \delta_4^{641} + \delta_4^{668} + \delta_4^{664} + \delta_4^{660} + \delta_4^{702} + \delta_4^{698} + \delta_4^{694} + \delta_4^{690} + \delta_4^{736} + \delta_4^{732} + \delta_4^{728} + \delta_4^{724} + \\ & \delta_4^{766} + \delta_4^{762} + \delta_4^{758} + \delta_4^{754} + \delta_4^{788} + \delta_4^{792} = 43 \end{aligned} \tag{1.c}$$

$$(1 - \delta_1^{656}) \leq 1 \tag{1.d}$$

$$(1 - \delta_2^{639}) \leq 1 \tag{1.e}$$

$$\begin{aligned} & (1-\delta_4^{45}) + (1-\delta_4^{74}) + (1-\delta_4^{40}) + (1-\delta_4^{70}) + (1-\delta_4^{96}) + (1-\delta_4^{100}) + (1-\delta_4^{36}) + \\ & (1-\delta_4^{68}) + (1-\delta_4^{132}) + (1-\delta_4^{128}) + (1-\delta_4^{163}) + (1-\delta_4^{193}) + (1-\delta_4^{227}) + (1-\delta_4^{257}) + \\ & (1-\delta_4^{291}) + (1-\delta_4^{321}) + (1-\delta_4^{365}) + (1-\delta_4^{395}) + (1-\delta_4^{429}) + (1-\delta_4^{459}) + (1-\delta_4^{493}) + \\ & (1-\delta_4^{523}) + (1-\delta_4^{557}) + (1-\delta_4^{587}) + (1-\delta_4^{611}) + (1-\delta_4^{641}) + (1-\delta_4^{668}) + (1-\delta_4^{664}) + \\ & (1-\delta_4^{660}) + (1-\delta_4^{702}) + (1-\delta_4^{698}) + (1-\delta_4^{694}) + (1-\delta_4^{690}) + (1-\delta_4^{736}) + (1-\delta_4^{732}) + \\ & (1-\delta_4^{728}) + (1-\delta_4^{724}) + (1-\delta_4^{766}) + (1-\delta_4^{762}) + (1-\delta_4^{758}) + (1-\delta_4^{754}) + (1-\delta_4^{788}) + \\ & (1-\delta_4^{792}) \leq 59 \end{aligned} \tag{1.f}$$

$$\delta_u^e + \delta_t^d \leq 1 \tag{1.g}$$

$$\delta_t^d \in \{0,1\} \tag{1.h}$$

$$z \geq 0 \tag{1.i}$$

Constraint (1.a) and (1.b) in Model (1) indicate that there are 1 piece each of first item and second item which positioned in board. Constraint (1.c) indicate that there are 43 pieces of item 4. Constraints (1.d), (1.e) and (1.f) limit the displacement between variables along the width. Constraints (1.g) indicate that each item placed on the board does not overlap one another. Constraints (1.h) indicate that each item is positioned on the board.

The improvement phase for the first pattern can be seen in Model (2).

Minimize (1)

Subject to (2)

(1.a), (1.b), (1.c), (1.g), (1.h), (1.i)

$$\delta_{tr}^d = 0 \tag{2.a}$$

Constraint (2.a) in Model (2) indicate that each item is positioned in the board.

The Compaction Phase for the first pattern can be seen in Model (3).

Minimize (1)

Subject to (3)

(1.a), (1.b), (1.c), (1.d), (1.e), (1.f), (1.g), (1.h), (1.i)

$$\begin{aligned} & \delta_1^{656} + \delta_2^{639} + \delta_4^{45} + \delta_4^{74} + \delta_4^{40} + \delta_4^{70} + \delta_4^{96} + \delta_4^{100} + \delta_4^{36} + \delta_4^{68} + \delta_4^{132} + \delta_4^{128} + \delta_4^{163} + \\ & \delta_4^{193} + \delta_4^{227} + \delta_4^{257} + \delta_4^{291} + \delta_4^{321} + \delta_4^{365} + \delta_4^{395} + \delta_4^{429} + \delta_4^{459} + \delta_4^{493} + \delta_4^{523} + \delta_4^{557} + \\ & \delta_4^{587} + \delta_4^{611} + \delta_4^{641} + \delta_4^{668} + \delta_4^{664} + \delta_4^{660} + \delta_4^{702} + \delta_4^{698} + \delta_4^{694} + \delta_4^{690} + \delta_4^{736} + \delta_4^{732} + \\ & \delta_4^{728} + \delta_4^{724} + \delta_4^{766} + \delta_4^{762} + \delta_4^{758} + \delta_4^{754} + \delta_4^{788} + \delta_4^{792} \geq 1 \end{aligned} \tag{3.a}$$

Constraints (3.a) indicate that each item placed on the board does not overlap one another.

3.2.2 3-Phase Matheuristic Model For The Second Pattern

The constructive phase for the second pattern can be seen in Model (4).

Minimize

$$z = 25.\delta_2^{416} + 26.\delta_2^{420} + 50.\delta_2^{816} + 4.\delta_4^{67} + 6.\delta_4^{97} + 8.\delta_4^{131} + 10.\delta_4^{161} + 12.\delta_4^{195} + 14.\delta_4^{225}$$
$$16.\delta_4^{259} + 18.\delta_4^{289} + 20.\delta_4^{333} + 22.\delta_4^{353} + 24.\delta_4^{387} + 26.\delta_4^{417} + 60.\delta_4^{483} + 32.\delta_4^{517} +$$
$$32.\delta_4^{513} + 68.\delta_4^{547} + 72.\delta_4^{581} + 36.\delta_4^{577} + 76.\delta_4^{615} + 76.\delta_4^{611} + 40.\delta_4^{641} + 80.\delta_4^{645} +$$
$$40.\delta_4^{649} + 84.\delta_4^{675} + 84.\delta_4^{679} + 44.\delta_4^{705} + 88\delta_4^{709} + 88.\delta_4^{713} + 92.\delta_4^{739} + 92.\delta_4^{743} +$$
$$92.\delta_4^{747} + 96.\delta_4^{773} + 96.\delta_4^{777}$$

Subject to (4)

$$\delta_2^{416} + \delta_2^{420} + \delta_2^{816} = 3 \qquad (4.a)$$
$$\delta_4^{67} + \delta_4^{97} + \delta_4^{131} + \delta_4^{161} + \delta_4^{195} + \delta_4^{225} + \delta_4^{259} + \delta_4^{289} + \delta_4^{333} + \delta_4^{353} + \delta_4^{387} + \delta_4^{417} + \delta_4^{483} +$$
$$\delta_4^{517} + \delta_4^{513} + \delta_4^{547} + \delta_4^{581} + \delta_4^{577} + \delta_4^{615} + \delta_4^{611} + \delta_4^{641} + \delta_4^{645} + \delta_4^{649} + \delta_4^{675} + \delta_4^{679} +$$
$$\delta_4^{705} + \delta_4^{709} + \delta_4^{713} + \delta_4^{739} + \delta_4^{743} + \delta_4^{747} + \delta_4^{773} + \delta_4^{777} = 33 \qquad (4.b)$$
$$(1-\delta_2^{416}) + (1-\delta_2^{420}) + (1-\delta_2^{816}) \leq 3 \qquad (4.c)$$
$$(1-\delta_4^{67}) + (1-\delta_4^{97}) + (1-\delta_4^{131}) + (1-\delta_4^{161}) + (1-\delta_4^{195}) + (1-\delta_4^{225}) + (1-\delta_4^{259}) +$$
$$(1-\delta_4^{289}) + (1-\delta_4^{333}) + (1-\delta_4^{353}) + (1-\delta_4^{387}) + (1-\delta_4^{417}) + (1-\delta_4^{483}) + (1-\delta_4^{517}) +$$
$$(1-\delta_4^{513}) + (1-\delta_4^{547}) + (1-\delta_4^{581}) + (1-\delta_4^{577}) + (1-\delta_4^{615}) + (1-\delta_4^{611}) + (1-\delta_4^{641}) +$$
$$(1-\delta_4^{645}) + (1-\delta_4^{649}) + (1-\delta_4^{675}) + (1-\delta_4^{679}) + (1-\delta_4^{705}) + (1-\delta_4^{709}) + (1-\delta_4^{713}) +$$
$$(1-\delta_4^{739}) + (1-\delta_4^{743}) + (1-\delta_4^{747}) + (1-\delta_4^{773}) + (1-\delta_4^{777}) \leq 48 \qquad (4.d)$$
$$\delta_u^e + \delta_t^d \leq 1 \qquad (4.e)$$
$$\delta_t^d \in \{0,1\} \qquad (4.f)$$
$$z \geq 0 \qquad (4.g)$$

Constraint (4.a) in Model (4) indicate that there are 3 pieces of second item which positioned in board. Constraint (4.b) indicate that there are 33 pieces of item 4. Constraints (4.c) and (4.d) limit the displacement between variables along the width. Constraints (4.e) indicate that each item placed on the board does not overlap one another. Constraints (4.f) indicate that each item is positioned on the board.

The improvement phase for the second pattern can be seen in Model (5).

Minimize (4)

Subject to (5)

$$(4.a), (4.b), (4.e), (4.f), (4.g)$$
$$\delta_{tr}^d = 0 \qquad (5.a)$$

Constraint (5.a) in Model (5) indicate that each item is positioned in the board.

The compaction phase for the second pattern can be seen in Model (6).

Minimize (4)

Subject to (6)

$$(4.a), (4.b), (4.e), (4.f), (4.g)$$
$$\delta_2^{416} + \delta_2^{420} + \delta_2^{816} + \delta_4^{67} + \delta_4^{97} + \delta_4^{131} + \delta_4^{161} + \delta_4^{195} + \delta_4^{225} + \delta_4^{259} + \delta_4^{289} + \delta_4^{333} + \delta_4^{353} + \delta_4^{387}$$
$$+\delta_4^{417} + \delta_4^{483} + \delta_4^{517} + \delta_4^{513} + \delta_4^{547} + \delta_4^{581} + \delta_4^{577} + \delta_4^{615} + \delta_4^{611} + \delta_4^{641} + \delta_4^{645} + \delta_4^{649} +$$
$$\delta_4^{675} + \delta_4^{679} + \delta_4^{705} + \delta_4^{709} + \delta_4^{713} + \delta_4^{739} + \delta_4^{743} + \delta_4^{747} + \delta_4^{773} + \delta_4^{777} \geq 1 \qquad (6.a)$$

Constraints (6.a) indicate that each item placed on the board does not overlap one another.

3.2.3 3-Phase Matheuristic Model For The Third Pattern

The constructive phase for the third pattern can be seen in Model (7).

Minimize

$$z = ((8.1)+0).\delta_3^{129} + ((20.1)+0).\delta_3^{166} + ((24.1)+0).\delta_3^{203} + ((14.1)+0).\delta_3^{240} +$$
$$((16.1)+0).\delta_3^{257} + ((36.1)+0).\delta_3^{294} + ((40.1)+0).\delta_3^{331} + ((22.1)+0).\delta_3^{368} + ((24.1)+$$
$$0).\delta_3^{385} + ((52.1)+0).\delta_3^{422} + ((56.1)+0).\delta_3^{459} + ((30.1)+0).\delta_3^{496} + ((32.1)+0).\delta_3^{513} +$$
$$((68.1)+0).\delta_3^{550} + ((72.1)+0).\delta_3^{587} + ((38.1)+0).\delta_3^{624} + ((40.1)+0).\delta_3^{641} + ((84.1)+$$
$$0).\delta_3^{678} + ((88.1)+0).\delta_3^{715} + ((46.1)+0).\delta_3^{752} + ((48.1)+0).\delta_3^{769} + ((100.1)+0).\delta_3^{806} +$$
$$((2.1)+0).\delta_4^{38} + ((4.1)+0).\delta_4^{42} + ((4.1)+0).\delta_4^{46} + ((4.1)+0)\delta_4^{76} + ((8.1)+0).\delta_4^{80} \qquad (7)$$

Subject to

$$\delta_3^{129} + \delta_3^{166} + \delta_3^{203} + \delta_3^{240} + \delta_3^{257} + \delta_3^{294} + \delta_3^{331} + \delta_3^{368} + \delta_3^{385} + \delta_3^{422} + \delta_3^{459} + \delta_3^{496} + \delta_3^{513} +$$
$$\delta_3^{550} + \delta_3^{587} + \delta_3^{624} + \delta_3^{641} + \delta_3^{678} + \delta_3^{715} + \delta_3^{752} + \delta_3^{769} + \delta_3^{806} = 22 \qquad (7.a)$$
$$\delta_4^{38} + \delta_4^{42} + \delta_4^{46} + \delta_4^{76} + \delta_4^{80} = 5 \qquad (7.b)$$
$$(1-\delta_3^{129}) + (1-\delta_3^{166}) + (1-\delta_3^{203}) + (1-\delta_3^{240}) + (1-\delta_3^{257}) + (1-\delta_3^{294}) + (1-\delta_3^{331}) +$$
$$(1-\delta_3^{368}) + (1-\delta_3^{385}) + (1-\delta_3^{422}) + (1-\delta_3^{459}) + (1-\delta_3^{496}) + (1-\delta_3^{513}) + (1-\delta_3^{550}) +$$
$$(1-\delta_3^{587}) + (1-\delta_3^{624}) + (1-\delta_3^{641}) + (1-\delta_3^{678}) + (1-\delta_3^{715}) + (1-\delta_3^{752}) + (1-\delta_3^{769}) +$$
$$(1-\delta_3^{806}) \leq 33 \qquad (7.c)$$
$$(1-\delta_4^{38}) + (1-\delta_4^{42}) + (1-\delta_4^{46}) + (1-\delta_4^{76}) + (1-\delta_4^{80}) \leq 8 \qquad (7.d)$$
$$\delta_u^e + \delta_t^d \leq 1 \qquad (7.e)$$
$$\delta_t^d \in \{0,1\} \qquad (7.f)$$
$$z \geq 0 \qquad (7.g)$$

Constraint (7.a) in Model (7) indicate that there are 22 pieces of third item which positioned in board. Constraint (7.b) indicate that there are 5 pieces of item 4. Constraints (7.c) and (7.d) limit the displacement between variables along the width. Constraints (7.e) indicate that each item placed on the board does not overlap one another. Constraints (7.f) indicate that each item is positioned on the board.

The improvement phase for the second pattern can be seen in Model (8).

Minimize (7)

Subject to (8)

$$(7.a), (7.b), (7.e), (7.f), (7.g)$$
$$\delta_{tr}^d = 0 \qquad (8.a)$$

Constraint (8.a) in Model (8) indicate that each item is positioned in the board. The compaction phase for the second pattern can be seen in Model (9).

Minimize (7)

Subject to (9)

$$(7.a), (7.b), (7.c), (7.d), (7.e), (7.f), (7.g)$$
$$\delta_3^{129} + \delta_3^{166} + \delta_3^{203} + \delta_3^{240} + \delta_3^{257} + \delta_3^{294} + \delta_3^{331} + \delta_3^{368} + \delta_3^{385} + \delta_3^{422} + \delta_3^{459} + \delta_3^{496} + \delta_3^{513} +$$
$$\delta_3^{550} + \delta_3^{587} + \delta_3^{624} + \delta_3^{641} + \delta_3^{678} + \delta_3^{715} + \delta_3^{752} + \delta_3^{769} + \delta_3^{806} + \delta_4^{38} + \delta_4^{42} + \delta_4^{46} + \delta_4^{76} +$$
$$\delta_4^{80} \geq 1 \qquad (9.a)$$

Constraints (9.a) indicate that each item placed on the board does not overlap one another. Based on the 3-Phase Matheuristic model, the minimum trim loss from the first pattern, second pattern and third pattern are 1,774 cm^2, 1749 cm^2, and 980 cm^2, respectively which used to minimize the use of stock length and width.

4. CONCLUSIONS

From the result and discussion, it can be concluded that 3 optimal cutting patterns were got from Modified Branch and Bound

Algorithm. All of the three patterns can be seen in Figure 1-3. The 3-Phase Matheuristic model is used to minimize the use of stock length and width. The minimum trim loss from the first pattern, second pattern and third pattern are 1,774 cm^2, 1749 cm^2, and 980 cm^2 respectively.

5. ACKNOWLEDGEMENT

This research is supported by Universitas Sriwijaya through Sains, Teknologi dan Seni (SATEKS) Research Grant Scheme, 2019.

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